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MAT 500

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**Chapter 4 Homework**

**4.2**

Let D be the outcome of the diagnostic test for a disease and be the true presence or absence of the disease. D = + represents a positive test result, D = - represents a negative test result, represents truly having the disease, and represents not having the disease.

For this particular test . In the general population, the probability of having the disease is , and this will be used as the prior for the first diagnostic test. We wish to find the probability that a person truly has the disease if he or she first receives a positive result and then is re-tested and receives a negative test result.

The posterior probability of truly having the disease after one positive test result is and this posterior will serve as the prior probability for the re-test. From Bayes rule we have that

Substituting , we have that the posterior probability of having the disease after a single positive test result is:

Now we will update the prior probability of truly having the disease to be , and then will be calculated to determine the posterior probability of truly having the disease given a negative retest. Again applying Bayes rule and the fact that

, it follows that

Therefore, the probability of truly having the disease after first receiving one positive and then receiving one negative test result is 0.000209.

**4.3**

Let D be the outcome of the diagnostic test for a disease and be the true presence or absence of the disease. D = + represents a positive test result, D = - represents a negative test result, represents truly having the disease, and represents truly not having the disease. The size of the population is n = 100000.

For this test . In the general population, the probability of having the disease is .

**(A)**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  | = **5094** |
|  |  |  | = **94906** |
|  |  |  | **n = 100000** |

**(B)**

Using the table from part A, the proportion of people who have the disease given a positive test result is This exactly matches the result found in problem 4.1.

**(C)**

n =**10000000**

(10000000)(0.001) = **10000**

(10000000)(0.999) = **9990000**

(9990000)(0.05) = **499500**

(10000)(0.99) = **9900**

(499500)(0.95) = **474525**

(9900)(0.01) = **99**

**(D)**

The proportion of people who test positive at first and then negative on retest and actually have the disease is This exactly matches the result from problem 4.2.

**4.4**

Let D be the outcome of the diagnostic test for a disease and be the true presence or absence of the disease. D = + represents a positive test result, D = - represents a negative test result, represents truly having the disease, and represents truly not having the disease.

For the diagnostic test we know that . In the general population, the probability of having the disease is , and this will be used as the prior for the first diagnostic test. We wish to find the probability that a person truly has the disease if he or she first receives a negative result, and then is re-tested and receives a positive test result. Applying Bayes rule, we have that the posterior probability of having the disease after a negative test result is

The posterior probability of truly having the disease after the first negative diagnostic test result is . This will be used as the prior probability of truly having the disease for the positive retest.

The posterior probability of truly having the disease after first receiving a negative result and then receiving a positive result is . This result is identical to the result obtained if the test result is first positive, and then negative, which was found in problem 4.2. This demonstrates that the order in which data are collected does not affect the final posterior probability.